Cambridge Assessment
International Education

Syllabus

Cambridge International AS & A Level Further Mathematics 9231

Use this syllabus for exams in 2026 and 2027.
Exams are available in the June and November series.

Version 1

For the purposes of screen readers, any mention in this document of Cambridge IGCSE refers to Cambridge International General Certificate of Secondary Education.

Cambridge Pathway

Why choose Cambridge International?

Cambridge International prepares school students for life, helping them develop an informed curiosity and a lasting passion for learning. We are part of the University of Cambridge.

Our Cambridge Pathway gives students a clear path for educational success from age 5 to 19. Schools can shape the curriculum around how they want students to learn – with a wide range of subjects and flexible ways to offer them. It helps students discover new abilities and a wider world, and gives them the skills they need for life, so they can achieve at school, university and work.

Our programmes and qualifications set the global standard for international education. They are created by subject experts, are rooted in academic rigour and reflect the latest educational research. They provide a strong platform for students to progress from one stage to the next, and are well supported by teaching and learning resources. Learn more about our research at **www.cambridgeassessment.org.uk/our-research/**

We review all our syllabuses regularly, so they reflect the latest research evidence and professional teaching practice – and take account of the different national contexts in which they are taught.

We consult with teachers to help us design each syllabus around the needs of their learners. Consulting with leading universities has helped us make sure our syllabuses encourage students to master the key concepts in the subject and develop the skills necessary for success in higher education.

We believe education works best when curriculum, teaching, learning and assessment are closely aligned. Our programmes develop deep knowledge, conceptual understanding and higher-order thinking skills, to prepare students for their future. Together with schools, we develop Cambridge learners who are confident, responsible, reflective, innovative and engaged – equipped for success in the modern world.

Every year, nearly a million Cambridge students from 10 000 schools in 160 countries prepare for their future with the Cambridge Pathway.

School feedback: 'We think the Cambridge curriculum is superb preparation for university.'

Feedback from: Christoph Guttentag, Dean of Undergraduate Admissions, Duke University, USA

Quality management

Cambridge International is committed to providing exceptional quality. In line with this commitment, our quality management system for the provision of international education programmes and qualifications programmes for students aged 5 to 19 is independently certified as meeting the internationally recognised standard, ISO 9001:2015. Learn more at **www.cambridgeinternational.org/about-us/our-standards/**

Contents

Important: Changes to this syllabus

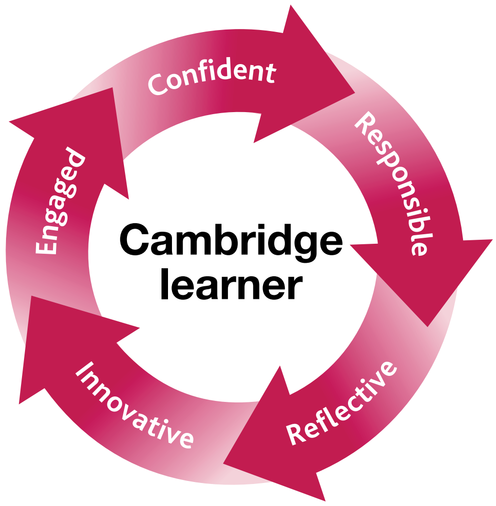
The latest syllabus is version 1, published September 2023. There are no significant changes which affect teaching.

Any textbooks endorsed to support the syllabus for examination from 2020 are still suitable for use with this syllabus.

1 Why choose this syllabus?

Key benefits

The best motivation for a student is a real passion for the subject they are learning. By offering students a variety of Cambridge International AS & A Levels, you can give them the greatest chance of finding the path of education they most want to follow. With over 50 subjects to choose from, students can select the ones they love and that they are best at, which helps motivate them throughout their studies.



Following a Cambridge International AS & A Level programme helps students develop abilities which universities value highly, including:

- a deep understanding of their subjects
- higher order thinking skills – analysis, critical thinking, problem solving
- presenting ordered and coherent arguments
- independent learning and research.

Cambridge International AS & A Level Further Mathematics develops a set of transferable skills. These include the skill of working with mathematical information, as well as the ability to think logically and independently, consider accuracy, model situations mathematically, analyse results and reflect on findings. Learners can apply these skills across a wide range of subjects and the skills equip them well for progression to higher education or directly into employment. Learners will find that the additional time spent studying this subject will support their understanding of A Level Mathematics.

Our approach in Cambridge International AS & A Level Further Mathematics encourages learners to be:

confident, using and sharing information and ideas, and using mathematical techniques to solve problems. These skills build confidence and support work in other subject areas as well as in mathematics.

responsible, through learning and applying skills which prepare them for future academic studies, helping them to become numerate members of society.

reflective, through making connections between different branches of mathematics and considering the outcomes of mathematical problems and modelling.

innovative, through solving both familiar and unfamiliar problems in different ways, selecting from a range of mathematical and problem-solving techniques.

engaged, by the beauty and structure of mathematics, its patterns and its many applications to real life situations.

School feedback: 'Cambridge students develop a deep understanding of subjects and independent thinking skills.'

Feedback from: Principal, Rockledge High School, USA

Key concepts

Key concepts are essential ideas that help students develop a deep understanding of their subject and make links between different aspects. Key concepts may open up new ways of thinking about, understanding or interpreting the important things to be learned.

Good teaching and learning will incorporate and reinforce a subject's key concepts to help students gain:

- a greater depth as well as breadth of subject knowledge
- confidence, especially in applying knowledge and skills in new situations
- the vocabulary to discuss their subject conceptually and show how different aspects link together
- a level of mastery of their subject to help them enter higher education.

The key concepts identified below, carefully introduced and developed, will help to underpin the course you will teach. You may identify additional key concepts which will also enrich teaching and learning.

The key concepts for Cambridge International AS & A Level Further Mathematics are:

- **Problem solving**
  Mathematics is fundamentally problem solving and representing systems and models in different ways. These include:
  - Algebra: this is an essential tool which supports and expresses mathematical reasoning and provides a means to generalise across a number of contexts.
  - Geometrical techniques: algebraic representations also describe a spatial relationship, which gives us a new way to understand a situation.
  - Calculus: this is a fundamental element which describes change in dynamic situations and underlines the links between functions and graphs.
  - Mechanical models: these explain and predict how particles and objects move or remain stable under the influence of forces.
  - Statistical methods: these are used to quantify and model aspects of the world around us. Probability theory predicts how chance events might proceed, and whether assumptions about chance are justified by evidence.

- **Communication**
  Mathematical proof and reasoning is expressed using algebra and notation so that others can follow each line of reasoning and confirm its completeness and accuracy. Mathematical notation is universal. Each solution is structured, but proof and problem solving also invite creative and original thinking.

- **Mathematical modelling**
  Mathematical modelling can be applied to many different situations and problems, leading to predictions and solutions. A variety of mathematical content areas and techniques may be required to create the model. Once the model has been created and applied, the results can be interpreted to give predictions and information about the real world.

International recognition and acceptance

Our expertise in curriculum, teaching and learning, and assessment is the basis for the recognition of our programmes and qualifications around the world. Every year thousands of students with Cambridge International AS & A Levels gain places at leading universities worldwide. Our programmes and qualifications are valued by top universities around the world including those in the UK, US (including Ivy League universities), Europe, Australia, Canada and New Zealand.

UK ENIC, the national agency in the UK for the recognition and comparison of international qualifications and skills, has carried out an independent benchmarking study of Cambridge International AS & A Level and found it to be comparable to the standard of AS & A Level in the UK. This means students can be confident that their Cambridge International AS & A Level qualifications are accepted as equivalent, grade for grade, to UK AS & A Levels by leading universities worldwide.

Cambridge International AS Level Further Mathematics makes up the first half of the Cambridge International A Level course in further mathematics and provides a foundation for the study of further mathematics at Cambridge International A Level. The AS Level can also be delivered as a standalone qualification. Depending on local university entrance requirements, students may be able to use it to progress directly to university courses in mathematics or some other subjects. It is also suitable as part of a course of general education.

Cambridge International A Level Mathematics provides a foundation for the study of mathematics or related courses in higher education. Equally it is suitable as part of a course of general education.

For more information about the relationship between the Cambridge International AS Level and Cambridge International A Level see the 'Assessment overview' section of the Syllabus overview.

We recommend learners check the Cambridge recognition database and university websites to find the most up-to-date entry requirements for courses they wish to study.

Learn more at **www.cambridgeinternational.org/recognition**

Supporting teachers

We believe education is most effective when curriculum, teaching and learning, and assessment are closely aligned. We provide a wide range of resources, detailed guidance, innovative training and targeted professional development so that you can give your students the best possible preparation for Cambridge International AS & A Level. To find out which resources are available for each syllabus go to **www.cambridgeinternational.org/support**

The School Support Hub is our secure online site for Cambridge teachers where you can find the resources you need to deliver our programmes. You can also keep up to date with your subject and the global Cambridge community through our online discussion forums.

Find out more at **www.cambridgeinternational.org/support**

Support for Cambridge International AS & A Level			
Planning and preparation • Syllabuses • Schemes of work • Specimen Question Papers and Mark Schemes • Teacher guides	**Teaching and assessment** • Endorsed resources • Online forums • Resource Plus	**Learning and revision** • Example candidate responses • Past papers and mark schemes • Specimen paper answers	**Results** • Candidate Results Service • Principal examiner reports for teachers

Sign up for email notifications about changes to syllabuses, including new and revised products and services, at **www.cambridgeinternational.org/syllabusupdates**

Syllabuses and specimen materials represent the final authority on the content and structure of all of our assessments.

Professional development

Find the next step on your professional development journey.

- Introductory Professional Development – An introduction to Cambridge programmes and qualifications.
- Extension Professional Development – Develop your understanding of Cambridge programmes and qualifications to build confidence in your delivery.
- Enrichment Professional Development – Transform your approach to teaching with our Enrichment workshops.
- Cambridge Professional Development Qualifications (PDQs) – Practice-based programmes that transform professional learning for practising teachers. Available at Certificate and Diploma level.

Find out more at:
www.cambridgeinternational.org/support-and-training-for-schools/professional-development/

Supporting exams officers

We provide comprehensive support and guidance for all Cambridge exams officers.
Find out more at: **www.cambridgeinternational.org/eoguide**

2 Syllabus overview

Aims

The aims describe the purposes of a course based on this syllabus.

The aims are to enable students to:

- further develop their mathematical knowledge and skills in a way which encourages confidence and provides satisfaction and enjoyment
- develop a greater understanding of mathematical principles and a further appreciation of mathematics as a logical and coherent subject
- acquire a greater range of mathematical skills, particularly those which will enable them to use applications of mathematics in the context of everyday situations and of other subjects they may be studying
- further develop the ability to analyse problems logically
- recognise when and how a situation may be represented mathematically, identify and interpret relevant factors and select an appropriate mathematical method to solve the problem
- use mathematics fluently as a means of communication with emphasis on the use of clear expression
- acquire the mathematical background necessary for further study in mathematics or related subjects.

Cambridge Assessment International Education is an education organisation and politically neutral. The contents of this syllabus, examination papers and associated materials do not endorse any political view. We endeavour to treat all aspects of the exam process neutrally.

Content overview

Content section	Assessment component	Topics included
1 Further Pure Mathematics 1	Paper 1	1.1 Roots of polynomial equations 1.2 Rational functions and graphs 1.3 Summation of series 1.4 Matrices 1.5 Polar coordinates 1.6 Vectors 1.7 Proof by induction
2 Further Pure Mathematics 2	Paper 2	2.1 Hyperbolic functions 2.2 Matrices 2.3 Differentiation 2.4 Integration 2.5 Complex numbers 2.6 Differential equations
3 Further Mechanics	Paper 3	3.1 Motion of a projectile 3.2 Equilibrium of a rigid body 3.3 Circular motion 3.4 Hooke's law 3.5 Linear motion under a variable force 3.6 Momentum
4 Further Probability & Statistics	Paper 4	4.1 Continuous random variables 4.2 Inference using normal and t-distributions 4.3 χ^2-tests 4.4 Non-parametric tests 4.5 Probability generating functions

School feedback: 'Cambridge International AS & A Levels prepare students well for university because they've learnt to go into a subject in considerable depth. There's that ability to really understand the depth and richness and the detail of a subject. It's a wonderful preparation for what they are going to face at university.'

Feedback from: US Higher Education Advisory Council

Structure

There are four components that can be combined in specific ways (please see below):

Paper 1: Further Pure Mathematics 1

Paper 2: Further Pure Mathematics 2

Paper 3: Further Mechanics

Paper 4: Further Probability & Statistics

All AS Level candidates take two written papers.

All A Level candidates take four written papers.

AS Level Further Mathematics

The Cambridge International AS Level Further Mathematics qualification offers two different options:

- Further Pure Mathematics 1 and Further Mechanics (Paper 1 and Paper 3)

  or

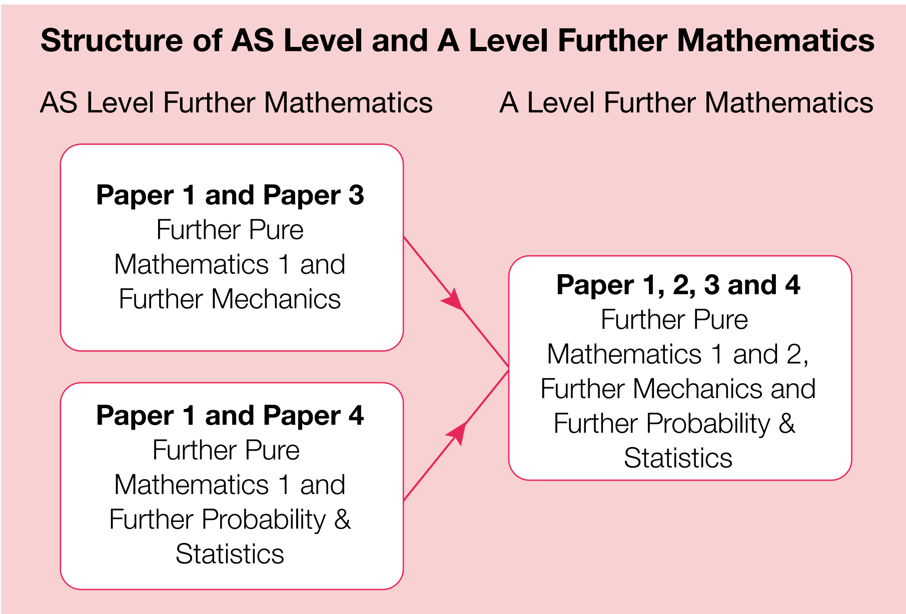
- Further Pure Mathematics 1 and Further Probability & Statistics (Paper 1 and Paper 4).

A Level Further Mathematics

Cambridge International A Level Further Mathematics includes all four components:

- Paper 1: Further Pure Mathematics 1
- Paper 2: Further Pure Mathematics 2
- Paper 3: Further Mechanics
- Paper 4: Further Probability & Statistics.

See page 10 for a table showing all possible assessment routes.



Please see the School Support Hub at **www.cambridgeinternational.org/support** for more information about parallel teaching of this syllabus alongside Cambridge International AS & A Level Mathematics and recommended routes through the Cambridge International AS & A Level Mathematics qualifications 9709 and 9231.

Assessment overview

Paper 1

Further Pure Mathematics 1 — 2 hours

75 marks

6 to 8 structured questions based on the Further Pure Mathematics 1 subject content

Answer all questions

Written examination

Externally assessed

60% of the AS Level

30% of the A Level

Compulsory for AS Level and A Level

Paper 3

Further Mechanics — 1 hour 30 minutes

50 marks

5 to 7 structured questions based on the Further Mechanics subject content

Answer all questions

Written examination

Externally assessed

40% of the AS Level

20% of the A Level

Offered as part of AS Level or A Level

Paper 2

Further Pure Mathematics 2 — 2 hours

75 marks

7 to 9 structured questions based on the Further Pure Mathematics 2 subject content

Answer all questions

Written examination

Externally assessed

30% of the A Level

Compulsory for A Level

Paper 4

Further Probability & Statistics — 1 hour 30 minutes

50 marks

5 to 7 structured questions based on the Further Probability & Statistics subject content

Answer all questions

Written examination

Externally assessed

40% of the AS Level

20% of the A Level

Offered as part of AS Level or A Level

Information on availability is in the **Before you start** section.

There are three routes for Cambridge International AS & A Level Mathematics:

Candidates may combine components as shown below to suit their particular interests.

Route 1 **AS Level only** (Candidates take the AS components in the same series)	**Paper 1** **Further Pure Mathematics 1**	**Paper 2** **Further Pure Mathematics 2**	**Paper 3** **Further Mechanics**	**Paper 4** **Further Probability & Statistics**
Either	yes	Not available for AS Level	yes	no
Or	yes		no	yes

Route 2 **A Level (staged over two years)**	**Paper 1** **Further Pure Mathematics 1**	**Paper 2** **Further Pure Mathematics 2**	**Paper 3** **Further Mechanics**	**Paper 4** **Further Probability & Statistics**
Either Year 1 AS Level*	yes	no	yes	no
Year 2 Complete the A Level	no	yes	no	yes
Or Year 1 AS Level*	yes	no	no	yes
Year 2 Complete the A Level	no	yes	yes	no

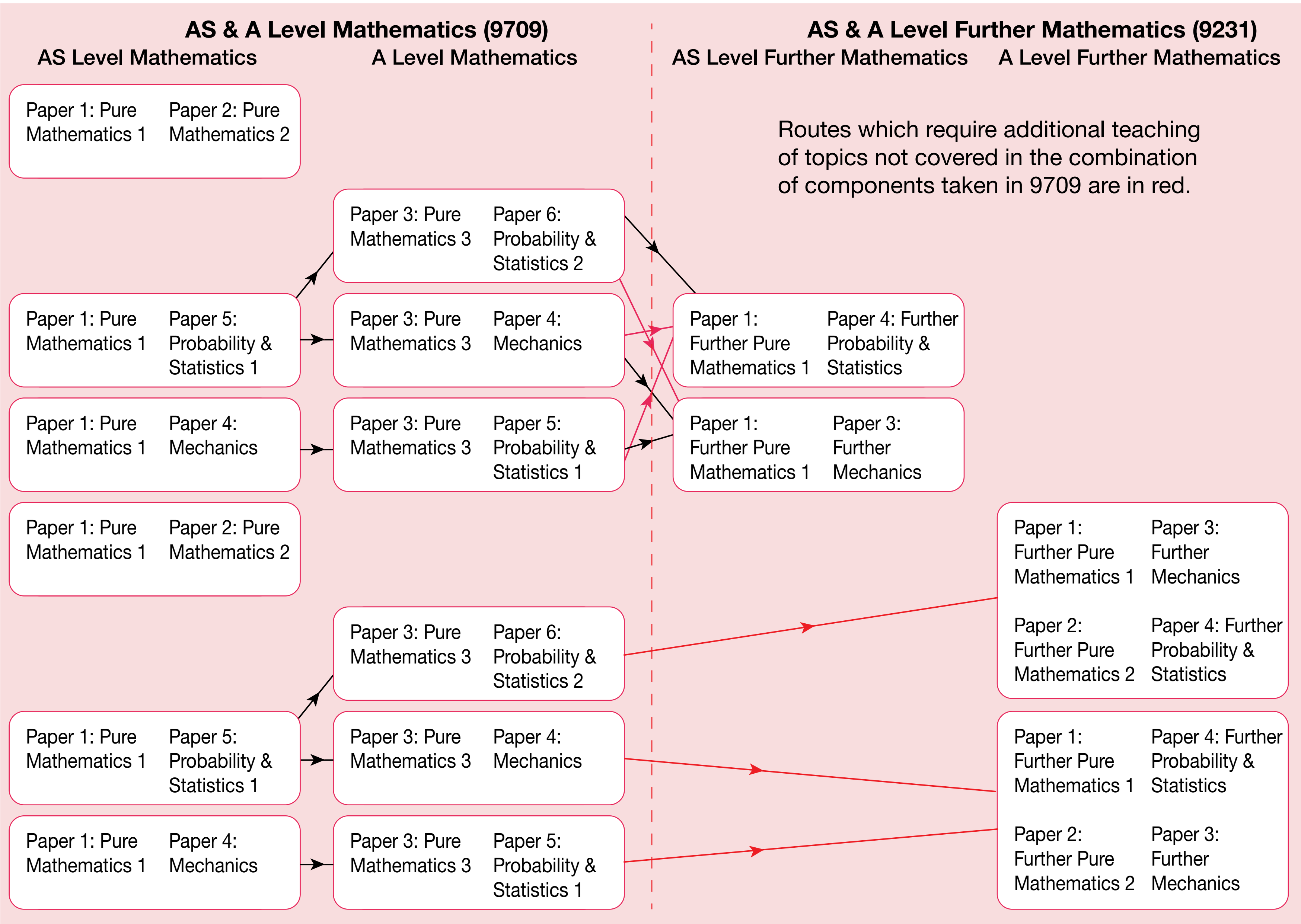
Route 3 **A Level** (Candidates take the A Level components in the same series)	**Paper 1** **Further Pure Mathematics 1**	**Paper 2** **Further Pure Mathematics 2**	**Paper 3** **Further Mechanics**	**Paper 4** **Further Probability & Statistics**
Year 2 full A Level	yes	yes	yes	yes

* Candidates carry forward their AS Level result subject to the rules and time limits described in the *Cambridge Handbook*. See **Making entries** for more information on carry forward of results [and marks].

Candidates following an AS Level route are eligible for grades a–e. Candidates following an A Level route are eligible for grades A*–E.

Back to contents page

Routes through AS & A Level Mathematics and AS & A Level Further Mathematics



Assessment objectives

The assessment objectives (AOs) are:

AO1 Knowledge and understanding

- Show understanding of relevant mathematical concepts, terminology and notation
- Recall accurately and use appropriate mathematical manipulative techniques

AO2 Application and communication

- Recognise the appropriate mathematical procedure for a given situation
- Apply appropriate combinations of mathematical skills and techniques in solving problems
- Present relevant mathematical work, and communicate corresponding conclusions, in a clear and logical way

Weighting for assessment objectives

The approximate weightings (± 5%) allocated to each of the assessment objectives (AOs) are summarised below.

Assessment objectives as a percentage of each qualification

Assessment objective	Weighting in AS Level %	Weighting in A Level %
AO1 Knowledge and understanding	45	45
AO2 Application and communication	55	55
Total	100	100

Assessment objectives as a percentage of each component

Assessment objective	Weighting in components %			
	Paper 1	Paper 2	Paper 3	Paper 4
AO1 Knowledge and understanding	45	45	45	45
AO2 Application and communication	55	55	55	55
Total	100	100	100	100

3 Subject content

This syllabus gives you the flexibility to design a course that will interest, challenge and engage your learners. Where appropriate you are responsible for selecting resources and examples to support your learners' study. These should be appropriate for the learners' age, cultural background and learning context as well as complying with your school policies and local legal requirements.

The mathematical content for each component is detailed below. You can teach the topics in any order you find appropriate. However, please note the prior knowledge requirements below, and the information about calculator use found in 4 Details of the assessment.

Notes and examples are included to clarify the syllabus content. Please note that these are examples only and examination questions may differ from the examples given.

Prior knowledge

It is expected that learners will have studied the majority of the Cambridge International AS & A Level Mathematics (9709) subject content before studying Cambridge International AS & A Level Further Mathematics (9231).

The prior knowledge required for each Further Mathematics component is shown in the following table.

Component in AS & A Level Further Mathematics (9231)	Prior knowledge required from AS & A Level Mathematics (9709)
9231 Paper 1: Further Pure Mathematics 1	9709 Papers 1 and 3
9231 Paper 2: Further Pure Mathematics 2	9709 Papers 1 and 3
9231 Paper 3: Further Mechanics	9709 Papers 1, 3 and 4
9231 Paper 4: Further Probability & Statistics	9709 Papers 1, 3, 5 and 6

Please see the School Support Hub at **www.cambridgeinternational.org/support** for more information about parallel teaching of Cambridge International AS & A Level Mathematics (9709) and AS & A Level Further Mathematics (9231).

The support document *Guide to prior learning for Paper 4 Further Probability & Statistics* can be found on the Cambridge International website.

1 Further Pure Mathematics 1 (for Paper 1)

1.1 Roots of polynomial equations

Candidates should be able to:	Notes and examples
• recall and use the relations between the roots and coefficients of polynomial equations	e.g. to evaluate symmetric functions of the roots or to solve problems involving unknown coefficients in equations; restricted to equations of degree 2, 3 or 4 only.
• use a substitution to obtain an equation whose roots are related in a simple way to those of the original equation	Substitutions will not be given for the easiest cases, e.g. where the new roots are reciprocals or squares or a simple linear function of the old roots.

1.2 Rational functions and graphs

Candidates should be able to:	Notes and examples
• sketch graphs of simple rational functions, including the determination of oblique asymptotes, in cases where the degree of the numerator and the demoninator are at most 2	Including determination of the set of values taken by the function, e.g. by the use of a discriminant. Detailed plotting of curves will not be required, but sketches will generally be expected to show significant features, such as turning points, asymptotes and intersections with the axes.
• understand and use relationships between the graphs of $y = \mathrm{f}(x)$, $y2 = \mathrm{f}(x)$, $y = \frac{1}{\mathrm{f}(x)}$, $y = \lvert \mathrm{f}(x) \rvert$ and $y = \mathrm{f}(\lvert x \rvert)$.	Including use of such sketch graphs in the course of solving equations or inequalities.

1.3 Summation of series

Candidates should be able to:	Notes and examples
• use the standard results for $\sum r$, $\sum r^2$, $\sum r^3$ to find related sums	
• use the method of differences to obtain the sum of a finite series	Use of partial fractions to express a general term in a suitable form may be required.
• recognise, by direct consideration of a sum to *n* terms, when a series is convergent, and find the sum to infinity in such cases	

1 Further Pure Mathematics 1

1.4 Matrices

Candidates should be able to:	Notes and examples
• carry out operations of matrix addition, subtraction and multiplication, and recognise the terms zero matrix and identity (or unit) matrix	Including non-square matrices. Matrices will have at most 3 rows and columns.
• recall the meaning of the terms 'singular' and 'non-singular' as applied to square matrices and, for 2×2 and 3×3 matrices, evaluate determinants and find inverses of non-singular matrices	The notations $\det \mathbf{M}$ for the determinant of a matrix $\mathbf{M}$, and $\mathbf{I}$ for the identity matrix, will be used.
• understand and use the result, for non-singular matrices, $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$	Extension to the product of more than two matrices may be required.
• understand the use of 2×2 matrices to represent certain geometric transformations in the x-y plane, in particular – understand the relationship between the transformations represented by $\mathbf{A}$ and $\mathbf{A}^{-1}$ – recognise that the matrix product $\mathbf{AB}$ represents the transformation that results from the transformation represented by $\mathbf{B}$ followed by the transformation represented by $\mathbf{A}$ – recall how the area scale factor of a transformation is related to the determinant of the corresponding matrix – find the matrix that represents a given transformation or sequence of transformations	Understanding of the terms 'rotation', 'reflection', 'enlargement', 'stretch' and 'shear' for 2D transformations will be required. Other 2D transformations may be included, but no particular knowledge of them is expected.
• understand the meaning of 'invariant' as applied to points and lines in the context of transformations represented by matrices, and solve simple problems involving invariant points and invariant lines.	e.g. to locate the invariant points of the transformation represented by $\begin{pmatrix} 6 & 5 \\ 2 & 3 \end{pmatrix}$, or to find the invariant lines through the origin for $\begin{pmatrix} 4 & -1 \\ 2 & 1 \end{pmatrix}$, or to show that any line with gradient 1 is invariant for $\begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}$.

1 Further Pure Mathematics 1

1.5 Polar coordinates

Candidates should be able to:	Notes and examples
• understand the relations between Cartesian and polar coordinates, and convert equations of curves from Cartesian to polar form and vice versa	The convention $r \geqslant 0$ will be used.
• sketch simple polar curves, for $0 \leqslant \theta < 2\pi$ or $-\pi < \theta \leqslant \pi$ or a subset of either of these intervals	Detailed plotting of curves will not be required, but sketches will generally be expected to show significant features, such as symmetry, coordinates of intersections with the initial line, the form of the curve at the pole and least/greatest values of r.
• recall the formula $\frac{1}{2}\int r^2 \mathrm{d}\theta$ for the area of a sector, and use this formula in simple cases.	

1.6 Vectors

Candidates should be able to:	Notes and examples
• use the equation of a plane in any of the forms $ax + by + cz = d$ or $\mathbf{r.n} = p$ or $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$ and convert equations of planes from one form to another as necessary in solving problems	
• recall that the vector product $\mathbf{a} \times \mathbf{b}$ of two vectors can be expressed either as $\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\sin\theta\,\hat{\mathbf{n}}$, where $\hat{\mathbf{n}}$ is a unit vector, or in component form as $(a_2b_3 - a_3b_2)\mathbf{i} + (a_3b_1 - a_1b_3)\mathbf{j} + (a_1b_2 - a_2b_1)\mathbf{k}$	
• use equations of lines and planes, together with scalar and vector products where appropriate, to solve problems concerning distances, angles and intersections, including – determining whether a line lies in a plane, is parallel to a plane or intersects a plane, and finding the point of intersection of a line and a plane when it exists – finding the foot of the perpendicular from a point to a plane – finding the angle between a line and a plane, and the angle between two planes – finding an equation for the line of intersection of two planes – calculating the shortest distance between two skew lines – finding an equation for the common perpendicular to two skew lines.	

1 Further Pure Mathematics 1

1.7 Proof by induction

Candidates should be able to:	Notes and examples
• use the method of mathematical induction to establish a given result	e.g. $\sum_{r=1}^{n} r^4 = \frac{1}{4}n^2(n+1)^2$, $u_n = \frac{1}{2}(1+3^{n-1})$ for the sequence given by $u_{n+1} = 3u_n - 1$ and $u_1 = 1$, $\begin{pmatrix} 4 & -1 \\ 6 & -1 \end{pmatrix}^n = \begin{pmatrix} 3\times 2^n - 2 & 1-2^n \\ 3\times 2^{n+1} - 6 & 3-2^{n+1} \end{pmatrix}$, $3^{2n} + 2\times 5^n - 3$ is divisible by 8.
• recognise situations where conjecture based on a limited trial followed by inductive proof is a useful strategy, and carry this out in simple cases.	e.g. find the nth derivative of $x\mathrm{e}^x$, find $\sum_{r=1}^{n} r \times r!$.

2 Further Pure Mathematics 2 (for Paper 2)

Knowledge of Paper 1: Further Pure Mathematics 1 subject content from this syllabus is assumed for this component.

2.1 Hyperbolic functions

Candidates should be able to:	Notes and examples
• understand the definitions of the hyperbolic functions $\sinh x$, $\cosh x$, $\tanh x$, $\operatorname{sech} x$, $\operatorname{cosech} x$, $\coth x$ in terms of the exponential function	
• sketch the graphs of hyperbolic functions	
• prove and use identities involving hyperbolic functions	e.g. $\cosh^2 x - \sinh^2 x \equiv 1$, $\sinh 2x \equiv 2\sinh x \cosh x$, and similar results corresponding to the standard trigonometric identities.
• understand and use the definitions of the inverse hyperbolic functions and derive and use the logarithmic forms	

2.2 Matrices

Candidates should be able to:	Notes and examples
• formulate a problem involving the solution of 3 linear simultaneous equations in 3 unknowns as a problem involving the solution of a matrix equation, or vice versa	
• understand the cases that may arise concerning the consistency or inconsistency of 3 linear simultaneous equations, relate them to the singularity or otherwise of the corresponding matrix, solve consistent systems, and interpret geometrically in terms of lines and planes	e.g. three planes meeting in a common point, or in a common line, or having no common points.
• understand the terms 'characteristic equation', 'eigenvalue' and 'eigenvector', as applied to square matrices	Including use of the definition $\mathbf{Ae} = \lambda\mathbf{e}$ to prove simple properties, e.g. that λ^n is an eigenvalue of $\mathbf{A}^n$.
• find eigenvalues and eigenvectors of 2×2 and 3×3 matrices	Restricted to cases where the eigenvalues are real and distinct.
• express a square matrix in the form $\mathbf{QDQ}^{-1}$, where $\mathbf{D}$ is a diagonal matrix of eigenvalues and $\mathbf{Q}$ is a matrix whose columns are eigenvectors, and use this expression	e.g. in calculating powers of 2×2 or 3×3 matrices.
• use the fact that a square matrix satisfies its own characteristic equation.	e.g. in finding successive powers of a matrix or finding an inverse matrix; restricted to 2×2 or 3×3 matrices only.

2 Further Pure Mathematics 2

2.3 Differentiation

Candidates should be able to:	Notes and examples
• differentiate hyperbolic functions and differentiate $\sin^{-1}x$, $\cos^{-1}x$, $\sinh^{-1}x$, $\cosh^{-1}x$ and $\tanh^{-1}x$	
• obtain an expression for $\frac{d^2y}{dx^2}$ in cases where the relation between x and y is defined implicitly or parametrically	
• derive and use the first few terms of a Maclaurin's series for a function.	Derivation of a general term is not included, but successive 'implicit' differentiation steps may be required, e.g. for $y = \tan x$ following an initial differentiation rearranged as $y' = 1 + y^2$.

2.4 Integration

Candidates should be able to:	Notes and examples
• integrate hyperbolic functions and recognise integrals of functions of the form $\frac{1}{\sqrt{a^2 - x^2}}$, $\frac{1}{\sqrt{x^2 + a^2}}$ and $\frac{1}{\sqrt{x^2 - a^2}}$, and integrate associated functions using trigonometric or hyperbolic substitutions as appropriate	Including use of completing the square where necessary, e.g. to integrate $\frac{1}{\sqrt{x^2 + x}}$.
• derive and use reduction formulae for the evaluation of definite integrals	e.g. $\int_0^{\frac{1}{2}\pi} \sin^n x \, dx$, $\int_0^1 e^{-x}(1 - x)^n \, dx$. In harder cases hints may be given, e.g. $\int_0^{\frac{1}{4}\pi} \sec^n x \, dx$ by considering $\frac{d}{dx}(\tan x \sec^n x)$.
• understand how the area under a curve may be approximated by areas of rectangles, and use rectangles to estimate or set bounds for the area under a curve or to derive inequalities or limits concerning sums	Questions may involve either rectangles of unit width or rectangles whose width can tend to zero, e.g. $1 + \ln n > \sum_{r=1}^{n} \frac{1}{r} > \ln(n + 1)$, $\sum_{r=1}^{n} \frac{1}{n}\left(1 + \frac{r}{n}\right)^{-1} \approx \int_0^1 (1 + x)^{-1} \, dx$.

continued

2 Further Pure Mathematics 2

2.4 Integration continued

Candidates should be able to:	Notes and examples
• use integration to find – arc lengths for curves with equations in Cartesian coordinates, including the use of a parameter, or in polar coordinates – surface areas of revolution about one of the axes for curves with equations in Cartesian coordinates, including the use of a parameter.	Any questions involving integration may require techniques from Cambridge International A Level Mathematics (9709) applied to more difficult cases, e.g. integration by parts for $\int e^x \sin x \, dx$, or use of the substitution $t = \tan \frac{1}{2}x$. Surface areas of revolution for curves with equations in polar coordinates will not be required.

2.5 Complex numbers

Candidates should be able to:	Notes and examples
• understand de Moivre's theorem, for a positive or negative integer exponent, in terms of the geometrical effect of multiplication and division of complex numbers	
• prove de Moivre's theorem for a positive integer exponent	e.g. by induction.
• use de Moivre's theorem for a positive or negative rational exponent – to express trigonometrical ratios of multiple angles in terms of powers of trigonometrical ratios of the fundamental angle – to express powers of $\sin\theta$ and $\cos\theta$ in terms of multiple angles – in the summation of series – in finding and using the nth roots of unity.	e.g. expressing $\cos 5\theta$ in terms of $\cos\theta$ or $\tan 5\theta$ in terms of $\tan\theta$. e.g. expressing $\sin^6\theta$ in terms of $\cos 2\theta$, $\cos 4\theta$ and $\cos 6\theta$. e.g. using the '$C + iS$' method to sum series such as $\sum_{r=1}^{n} \binom{n}{r} \sin r\theta$.

2.6 Differential equations

Candidates should be able to:	Notes and examples
• find an integrating factor for a first order linear differential equation, and use an integrating factor to find the general solution	e.g. $\frac{dy}{dx} - 2y = x^2$, $x\frac{dy}{dx} - y = x^4$, $\frac{dy}{dx} + y \coth x = \cosh x$.
• recall the meaning of the terms 'complementary function' and 'particular integral' in the context of linear differential equations, and recall that the general solution is the sum of the complementary function and a particular integral	

continued

2 Further Pure Mathematics 2

2.6 Differential equations continued

Candidates should be able to:	Notes and examples
• find the complementary function for a first or second order linear differential equation with constant coefficients	For second order equations, including the cases where the auxiliary equation has distinct real roots, a repeated real root or conjugate complex roots.
• recall the form of, and find, a particular integral for a first or second order linear differential equation in the cases where a polynomial or $a\,\mathrm{e}bx$ or $a\cos px + b\sin px$ is a suitable form, and in other simple cases find the appropriate coefficient(s) given a suitable form of particular integral	e.g. evaluate k given that $kx\cos 2x$ is a particular integral of $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 4y = \sin 2x$.
• use a given substitution to reduce a differential equation to a first or second order linear equation with constant coefficients or to a first order equation with separable variables	e.g. the substitution $x = \mathrm{e}t$ to reduce to linear form a differential equation with terms of the form $ax^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + bx\frac{\mathrm{d}y}{\mathrm{d}x} + cy$, or the substitution $y = ux$ to reduce $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x+y}{x-y}$ to separable form.
• use initial conditions to find a particular solution to a differential equation, and interpret a solution in terms of a problem modelled by a differential equation.	

3 Further Mechanics (for Paper 3)

Knowledge of Cambridge International AS & A Level Mathematics (9709) Paper 4: Mechanics subject content is assumed for this component.

3.1 Motion of a projectile

Candidates should be able to:	Notes and examples
• model the motion of a projectile as a particle moving with constant acceleration and understand any limitations of the model	Vector methods are not required
• use horizontal and vertical equations of motion to solve problems on the motion of projectiles, including finding the magnitude and direction of the velocity at a given time or position, the range on a horizontal plane and the greatest height reached	
• derive and use the Cartesian equation of the trajectory of a projectile, including problems in which the initial speed and/or angle of projection may be unknown.	Knowledge of the 'bounding parabola' for accessible points is not included.

3.2 Equilibrium of a rigid body

Candidates should be able to:	Notes and examples
• calculate the moment of a force about a point	For questions involving coplanar forces only; understanding of the vector nature of moments is not required.
• use the result that the effect of gravity on a rigid body is equivalent to a single force acting at the centre of mass of the body, and identify the position of the centre of mass of a uniform body using considerations of symmetry	
• use given information about the position of the centre of mass of a triangular lamina and other simple shapes	Proofs of results given in the MF19 List of formulae are not required.
• determine the position of the centre of mass of a composite body by considering an equivalent system of particles	Simple cases only, e.g. a uniform L-shaped lamina, or a uniform cone joined at its base to a uniform hemisphere of the same radius.
• use the principle that if a rigid body is in equilibrium under the action of coplanar forces then the vector sum of the forces is zero and the sum of the moments of the forces about any point is zero, and the converse of this	
• solve problems involving the equilibrium of a single rigid body under the action of coplanar forces, including those involving toppling or sliding.	

3 Further Mechanics

3.3 Circular motion

Candidates should be able to:	Notes and examples
• understand the concept of angular speed for a particle moving in a circle, and use the relation $v = r\omega$	
• understand that the acceleration of a particle moving in a circle with constant speed is directed towards the centre of the circle, and use the formulae $r\omega^2$ and $\frac{v^2}{r}$.	Proof of the acceleration formulae is not required.
• solve problems which can be modelled by the motion of a particle moving in a horizontal circle with constant speed	
• solve problems which can be modelled by the motion of a particle in a vertical circle without loss of energy.	Including finding a normal contact force or the tension in a string, locating points at which these are zero, and condidtions for complete circular motion.

3.4 Hooke's law

Candidates should be able to:	Notes and examples
• use Hooke's law as a model relating the force in an elastic string or spring to the extension or compression, and understand the term modulus of elasticity	
• use the formula for the elastic potential energy stored in a string or spring	Proof of the formula is not required.
• solve problems involving forces due to elastic strings or springs, including those where considerations of work and energy are needed.	e.g. a particle moving horizontally or vertically or on an inclined plane while attached to one or more strings or springs, or a particle attached to an elastic string acting as a 'conical pendulum'.

3.5 Linear motion under a variable force

Candidates should be able to:	Notes and examples
• solve problems which can be modelled as the linear motion of a particle under the action of a variable force, by setting up and solving an appropriate differential equation.	Including use of $v\frac{\mathrm{d}v}{\mathrm{d}x}$ for acceleration, where appropriate. Calculus required is restricted to content from Pure Mathematics 3 in Cambridge International A Level Mathematics (9709). Only differential equations in which the variables are separable are included.

3 Further Mechanics

3.6 Momentum

Candidates should be able to:	Notes and examples
• recall Newton's experimental law and the definition of the coefficient of restitution, the property $0 \leqslant e \leqslant 1$, and the meaning of the terms 'perfectly elastic' $(e = 1)$ and 'inelastic' $(e = 0)$ • use conservation of linear momentum and/or Newton's experimental law to solve problems that may be modelled as the direct or oblique impact of two smooth spheres, or the direct or oblique impact of a smooth sphere with a fixed surface.	

4 Further Probability & Statistics (for Paper 4)

Knowledge of Cambridge International AS & A Level Mathematics (9709) Papers 5 and 6: Probability & Statistics subject content is assumed for this component.

Please see the support document *Guide to prior learning for Paper 4 Further Probability & Statistics* on the Cambridge website for recommended prior knowledge for this paper.

4.1 Continuous random variables

Candidates should be able to:	Notes and examples
• use a probability density function which may be defined piecewise	
• use the general result $\mathrm{E}(\mathrm{g}(X)) = \int \mathrm{f}(x)\mathrm{g}(x)\,\mathrm{d}x$ where $\mathrm{f}(x)$ is the probability density function of the continuous random variable X and $\mathrm{g}(X)$ is a function of X	
• understand and use the relationship between the probability density function (PDF) and the cumulative distribution function (CDF), and use either to evaluate probabilities or percentiles	
• use cumulative distribution functions (CDFs) of related variables in simple cases.	e.g. given the CDF of a variable X, find the CDF of a related variable Y, and hence its PDF, e.g. where $Y = X^3$.

4.2 Inference using normal and *t*-distributions

Candidates should be able to:	Notes and examples
• formulate hypotheses and apply a hypothesis test concerning the population mean using a small sample drawn from a normal population of unknown variance, using a *t*-test	
• calculate a pooled estimate of a population variance from two samples	Calculations based on either raw or summarised data may be required.
• formulate hypotheses concerning the difference of population means, and apply, as appropriate – a 2-sample *t*-test – a paired sample *t*-test – a test using a normal distribution	The ability to select the test appropriate to the circumstances of a problem is expected.
• determine a confidence interval for a population mean, based on a small sample from a normal population with unknown variance, using a *t*-distribution	
• determine a confidence interval for a difference of population means, using a *t*-distribution or a normal distribution, as appropriate.	

4 Further Probability & Statistics

4.3 χ^2-tests

Candidates should be able to:	Notes and examples
• fit a theoretical distribution, as prescribed by a given hypothesis, to given data	Questions will not involve lengthy calculations.
• use a χ^2-test, with the appropriate number of degrees of freedom, to carry out the corresponding goodness of fit analysis	Classes should be combined so that each expected frequency is at least 5.
• use a χ^2-test, with the appropriate number of degrees of freedom, for independence in a contingency table.	Yates' correction is not required. Where appropriate, either rows or columns should be combined so that the expected frequency in each cell is at least 5.

4.4 Non-parametric tests

Candidates should be able to:	Notes and examples
• understand the idea of a non-parametric test and appreciate situations in which such a test might be useful	e.g. when sampling from a population which cannot be assumed to be normally distributed.
• understand the basis of the sign test, the Wilcoxon signed-rank test and the Wilcoxon rank-sum test	Including knowledge that Wilcoxon tests are valid only for symmetrical distributions.
• use a single-sample sign test and a single-sample Wilcoxon signed-rank test to test a hypothesis concerning a population median	Including the use of normal approximations where appropriate. Questions will not involve tied ranks or observations equal to the population median value being tested.
• use a paired-sample sign test, a Wilcoxon matched-pairs signed-rank test and a Wilcoxon rank-sum test, as appropriate, to test for identity of populations.	Including the use of normal approximations where appropriate. Questions will not involve tied ranks or zero-difference pairs.

4.5 Probability generating functions

Candidates should be able to:	Notes and examples
• understand the concept of a probability generating function (PGF) and construct and use the PGF for given distributions	Including the discrete uniform, binomial, geometric and Poisson distributions.
• use formulae for the mean and variance of a discrete random variable in terms of its PGF, and use these formulae to calculate the mean and variance of a given probability distribution	
• use the result that the PGF of the sum of independent random variables is the product of the PGFs of those random variables.	

4 Details of the assessment

Planning for assessment of AS & A Level Further Mathematics (9231)

S & A Level Further Mathematics (9231) can be studied with AS & A Level Mathematics (9709). However, learners may have completed their course in AS & A Level Mathematics before starting the Further Mathematics course.

The AS Level syllabuses are not designed for taking AS Mathematics (9709) and AS Further Mathematics (9231) in parallel after only one year of study. This is because AS Further Mathematics (9231) topics may depend on assumed prior knowledge from A Level Mathematics (9709), such as the Paper 3: Pure Mathematics 3 subject content. Therefore you must plan your teaching carefully if you intend to enter candidates for AS Level Further Mathematics after one year.

See the introduction to section 3 of this syllabus for a summary of the assumed prior knowledge required for AS & A Level Further Mathematics (9231).

Examination information

All components are assessed by written examinations which are externally marked. Sample assessment materials are available on our website at **www.cambridgeinternational.org** showing the question style and level of the examination papers.

Application of mathematical techniques

As well as demonstrating the appropriate techniques, candidates need to apply their knowledge in solving problems. Individual examination questions may involve ideas and methods from more than one section of the syllabus content for that component.

The main focus of examination questions will be the AS & A Level Further Mathematics syllabus content. However, candidates may need to make use of prior knowledge and mathematical techniques from previous study, as listed in the introduction to section 3 of this syllabus.

Structure of the question paper

All questions in the examination papers are compulsory. An approximate number of questions for each paper is given in the Assessment overview in section 2 of this syllabus. Questions are of varied lengths and often contain several parts, labelled (a), (b), (c), which may have sub-parts (i), (ii), (iii), as needed. Some questions might require candidates to sketch graphs or diagrams, or draw accurate graphs.

Answer space

Candidates answer on the question paper. All working should be shown neatly and clearly in the spaces provided for each question. New questions often start on a fresh page, so more answer space may be provided than is needed. If additional space is required, candidates should use the lined page at the end of the question paper, where the question number or numbers must be clearly shown.

Degrees of accuracy

Candidates should give non-exact numerical answers correct to three significant figures (or one decimal place for angles in degrees) unless a different level of accuracy is specified in the question. To earn accuracy marks, candidates should avoid rounding figures until they have their final answer.

Additional materials for examinations

Candidates are expected to have the following equipment in examinations:

- a ruler
- a scientific calculator (see the following section).

Note: a protractor and a pair of compasses are not required.

A list of formulae and statistical tables (MF19) is supplied in examinations for the use of candidates. A copy of the list of formulae and tables is given for reference in section 5 of this syllabus. Note that MF19 is a combined formulae list for AS & A Level Mathematics (9709) and AS & A Level Further Mathematics (9231). All formulae in the list may be needed for this syllabus.

Calculators

It is expected that candidates will have a calculator with standard 'scientific' functions available for use in all the examinations. Computers, graphical calculators and calculators capable of symbolic algebraic manipulation or symbolic differentiation or integration are not permitted. The General Regulations concerning the use of calculators are contained in the *Cambridge Handbook* at **www.cambridgeinternational.org/examsofficers**

Candidates are expected to show all necessary working; no marks will be given for unsupported answers from a calculator.

Mathematical notation

The list of mathematical notation that may be used in examinations for this syllabus is available on our website at **www.cambridgeinternational.org/9231**

Command words

Command words and their meanings help candidates know what is expected from them in the exams. The table below includes command words used in the assessment for this syllabus. The use of the command word will relate to the subject context.

Command word	What it means
Calculate	work out from given facts, figures or information
Deduce	conclude from available information
Derive	obtain something (expression/equation/value) from another by a sequence of logical steps
Describe	state the points of a topic / give characteristics and main features
Determine	establish with certainty
Evaluate	judge or calculate the quality, importance, amount, or value of something
Explain	set out purposes or reasons / make the relationships between things clear / say why and/or how and support with relevant evidence
Identify	name/select/recognise
Interpret	identify meaning or significance in relation to the context
Justify	support a case with evidence/argument
Prove	confirm the truth of the given statement using a chain of logical mathematical reasoning
Show (that)	provide structured evidence that leads to a given resul
Sketch	make a simple freehand drawing showing the key features, taking care over proportions
State	express in clear terms
Verify	Confim a given statement/result is true

5 List of formulae and statistical tables (MF19)

PURE MATHEMATICS

Mensuration

Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$

Volumc of conc or pyramid = $\frac{1}{3} \times$ basc arca $\times$ hcight

Area of curved surface of cone = $\pi r \times$ slant height

Arc length of circle $= r\theta$ (θ in radians)

Area of sector of circle $= \frac{1}{2}r^2\theta$ (θ in radians)

Algebra

For the quadratic equation $ax^2 + bx + c = 0$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For an arithmetic series:

$$u_n = a + (n-1)d\,, \qquad S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n\{2a + (n-1)d\}$$

For a geometric series:

$$u_n = ar^{n-1}\,, \qquad S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)\,, \qquad S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

Binomial series:

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \ldots + b^n\text{, where } n \text{ is a positive integer}$$

$$\text{and } \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots\text{, where } n \text{ is rational and } |x| < 1$$

Trigonometry

$$\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$$

$$\cos^2\theta + \sin^2\theta \equiv 1, \qquad 1 + \tan^2\theta \equiv \sec^2\theta, \qquad \cot^2\theta + 1 \equiv \operatorname{cosec}^2\theta$$

$$\sin(A \pm B) \equiv \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) \equiv \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

$$\sin 2A \equiv 2\sin A\cos A$$

$$\cos 2A \equiv \cos^2 A - \sin^2 A \equiv 2\cos^2 A - 1 \equiv 1 - 2\sin^2 A$$

$$\tan 2A \equiv \frac{2\tan A}{1 - \tan^2 A}$$

Principal values:

$$-\tfrac{1}{2}\pi \leqslant \sin^{-1} x \leqslant \tfrac{1}{2}\pi, \qquad 0 \leqslant \cos^{-1} x \leqslant \pi, \qquad -\tfrac{1}{2}\pi < \tan^{-1} x < \tfrac{1}{2}\pi$$

Differentiation

$\mathbf{f}(x)$	$\mathbf{f}'(x)$
x^n	nx^{n-1}
$\ln x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x\tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x\cot x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
uv	$v\frac{du}{dx} + u\frac{dv}{dx}$
$\frac{u}{v}$	$\frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

If $x = f(t)$ and $y = g(t)$ then $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$

Integration

(Arbitrary constants are omitted; a denotes a positive constant.)

$\mathbf{f}(x)$	$\int \mathbf{f}(x)\,\mathbf{d}x$	
x^n	$\dfrac{x^{n+1}}{n+1}$	$(n \neq -1)$
$\dfrac{1}{x}$	$\ln\lvert x \rvert$	
e^x	e^x	
$\sin x$	$-\cos x$	
$\cos x$	$\sin x$	
$\sec^2 x$	$\tan x$	
$\dfrac{1}{x^2+a^2}$	$\dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right)$	
$\dfrac{1}{x^2-a^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{x-a}{x+a}\right\rvert$	$(x > a)$
$\dfrac{1}{a^2-x^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{a+x}{a-x}\right\rvert$	$(\lvert x \rvert < a)$

$$\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$$

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln\lvert \mathrm{f}(x)\rvert$$

Vectors

If $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ then

$$\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3 = \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta$$

FURTHER PURE MATHEMATICS

Algebra

Summations:

$$\sum_{r=1}^{n} r = \tfrac{1}{2}n(n+1)\,, \qquad \sum_{r=1}^{n} r^2 = \tfrac{1}{6}n(n+1)(2n+1)\,, \qquad \sum_{r=1}^{n} r^3 = \tfrac{1}{4}n^2(n+1)^2$$

Maclaurin's series:

$$\mathrm{f}(x) = \mathrm{f}(0) + x\,\mathrm{f}'(0) + \frac{x^2}{2!}\mathrm{f}''(0) + \ldots + \frac{x^r}{r!}\mathrm{f}^{(r)}(0) + \ldots$$

$$\mathrm{e}^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \ldots + \frac{x^r}{r!} + \ldots \qquad \text{(all } x\text{)}$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + (-1)^{r+1}\frac{x^r}{r} + \ldots \qquad (-1 < x \leqslant 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \ldots \qquad \text{(all } x\text{)}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + (-1)^r \frac{x^{2r}}{(2r)!} + \ldots \qquad \text{(all } x\text{)}$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots + (-1)^r \frac{x^{2r+1}}{2r+1} + \ldots \qquad (-1 \leqslant x \leqslant 1)$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots + \frac{x^{2r+1}}{(2r+1)!} + \ldots \qquad \text{(all } x\text{)}$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots + \frac{x^{2r}}{(2r)!} + \ldots \qquad \text{(all } x\text{)}$$

$$\tanh^{-1} x = x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots + \frac{x^{2r+1}}{2r+1} + \ldots \qquad (-1 < x < 1)$$

Trigonometry

If $t = \tan\tfrac{1}{2}x$ then:

$$\sin x = \frac{2t}{1+t^2} \qquad \text{and} \qquad \cos x = \frac{1-t^2}{1+t^2}$$

Hyperbolic functions

$$\cosh^2 x - \sinh^2 x \equiv 1\,, \qquad \sinh 2x \equiv 2\sinh x \cosh x\,, \qquad \cosh 2x \equiv \cosh^2 x + \sinh^2 x$$

$$\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)$$

$$\cosh^{-1} x = \ln\left(x + \sqrt{x^2-1}\right) \qquad (x \geqslant 1)$$

$$\tanh^{-1} x = \tfrac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \qquad (|x| < 1)$$

Differentiation

$\mathbf{f}(x)$	$\mathbf{f}'(x)$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\operatorname{sech}^2 x$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cosh^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$

Integration

(Arbitrary constants are omitted; a denotes a positive constant.)

$\mathbf{f}(x)$	$\int \mathbf{f}(x)\,\mathbf{d}x$	
$\sec x$	$\ln\lvert\sec x + \tan x\rvert = \ln\lvert\tan(\frac{1}{2}x + \frac{1}{4}\pi)\rvert$	$(\lvert x\rvert < \frac{1}{2}\pi)$
$\operatorname{cosec} x$	$-\ln\lvert\operatorname{cosec} x + \cot x\rvert = \ln\lvert\tan(\frac{1}{2}x)\rvert$	$(0 < x < \pi)$
$\sinh x$	$\cosh x$	
$\cosh x$	$\sinh x$	
$\operatorname{sech}^2 x$	$\tanh x$	
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\frac{x}{a}\right)$	$(\lvert x\rvert < a)$
$\frac{1}{\sqrt{x^2-a^2}}$	$\cosh^{-1}\left(\frac{x}{a}\right)$	$(x > a)$
$\frac{1}{\sqrt{a^2+x^2}}$	$\sinh^{-1}\left(\frac{x}{a}\right)$	

MECHANICS

Uniformly accelerated motion

$$v = u + at, \qquad s = \tfrac{1}{2}(u + v)t, \qquad s = ut + \tfrac{1}{2}at^2, \qquad v^2 = u^2 + 2as$$

FURTHER MECHANICS

Motion of a projectile

Equation of trajectory is:

$$y = x\tan\theta - \frac{gx^2}{2V^2\cos^2\theta}$$

Elastic strings and springs

$$T = \frac{\lambda x}{l}, \qquad E = \frac{\lambda x^2}{2l}$$

Motion in a circle

For uniform circular motion, the acceleration is directed towards the centre and has magnitude

$$\omega^2 r \quad \text{or} \quad \frac{v^2}{r}$$

Centres of mass of uniform bodies

Triangular lamina: $\tfrac{2}{3}$ along median from vertex

Solid hemisphere of radius r: $\tfrac{3}{8}r$ from centre

Hemispherical shell of radius r: $\tfrac{1}{2}r$ from centre

Circular arc of radius r and angle 2α: $\dfrac{r\sin\alpha}{\alpha}$ from centre

Circular sector of radius r and angle 2α: $\dfrac{2r\sin\alpha}{3\alpha}$ from centre

Solid cone or pyramid of height h: $\tfrac{3}{4}h$ from vertex

PROBABILITY & STATISTICS

Summary statistics

For ungrouped data:

$$\bar{x} = \frac{\Sigma x}{n}, \qquad \text{standard deviation} = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2}$$

For grouped data:

$$\bar{x} = \frac{\Sigma xf}{\Sigma f}, \qquad \text{standard deviation} = \sqrt{\frac{\Sigma(x-\bar{x})^2 f}{\Sigma f}} = \sqrt{\frac{\Sigma x^2 f}{\Sigma f} - \bar{x}^2}$$

Discrete random variables

$$\mathrm{E}(X) = \Sigma xp, \qquad \mathrm{Var}(X) = \Sigma x^2 p - \{\mathrm{E}(X)\}^2$$

For the binomial distribution $\mathrm{B}(n, p)$:

$$p_r = \binom{n}{r} p^r (1-p)^{n-r}, \qquad \mu = np, \qquad \sigma^2 = np(1-p)$$

For the geometric distribution Geo(p):

$$p_r = p(1-p)^{r-1}, \qquad \mu = \frac{1}{p}$$

For the Poisson distribution $\mathrm{Po}(\lambda)$

$$p_r = \mathrm{e}^{-\lambda} \frac{\lambda^r}{r!}, \qquad \mu = \lambda, \qquad \sigma^2 = \lambda$$

Continuous random variables

$$\mathrm{E}(X) = \int x\,\mathrm{f}(x)\,\mathrm{d}x, \qquad \mathrm{Var}(X) = \int x^2\,\mathrm{f}(x)\,\mathrm{d}x - \{\mathrm{E}(X)\}^2$$

Sampling and testing

Unbiased estimators:

$$\bar{x} = \frac{\Sigma x}{n}, \qquad s^2 = \frac{\Sigma(x-\bar{x})^2}{n-1} = \frac{1}{n-1}\left(\Sigma x^2 - \frac{(\Sigma x)^2}{n}\right)$$

Central Limit Theorem:

$$\bar{X} \sim \mathrm{N}\left(\mu, \frac{\sigma^2}{n}\right)$$

Approximate distribution of sample proportion:

$$\mathrm{N}\left(p, \frac{p(1-p)}{n}\right)$$

FURTHER PROBABILITY & STATISTICS

Sampling and testing

Two-sample estimate of a common variance:

$$s^2 = \frac{\Sigma(x_1 - \bar{x}_1)^2 + \Sigma(x_2 - \bar{x}_2)^2}{n_1 + n_2 - 2}$$

Probability generating functions

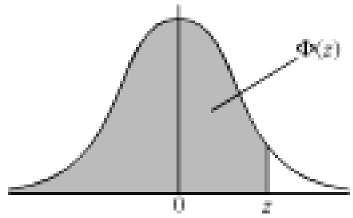
$$\mathrm{G}_X(t) = \mathrm{E}(t^X), \qquad \mathrm{E}(X) = \mathrm{G}'_X(1), \qquad \mathrm{Var}(X) = \mathrm{G}''_X(1) + \mathrm{G}'_X(1) - \{\mathrm{G}'_X(1)\}^2$$

THE NORMAL DISTRIBUTION FUNCTION

If Z has a normal distribution with mean 0 and variance 1, then, for each value of z, the table gives the value of $\Phi(z)$, where

$$\Phi(z) = \mathrm{P}(Z \leqslant z).$$

For negative values of z, use $\Phi(-z) = 1 - \Phi(z)$.



z	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
															ADD				
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	4	8	12	16	20	24	28	32	36
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4	8	12	16	20	24	28	32	36
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4	8	12	15	19	23	27	31	35
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4	7	11	15	19	22	26	30	34
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	4	7	11	14	18	22	25	29	32
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3	7	10	14	17	20	24	27	31
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	7	10	13	16	19	23	26	29
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3	6	9	12	15	18	21	24	27
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	3	5	8	11	14	16	19	22	25
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	2	5	7	9	12	14	16	19	21
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	2	4	6	8	10	12	14	16	18
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015	2	4	6	7	9	11	13	15	17
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	2	3	5	6	8	10	11	13	14
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	1	3	4	6	7	8	10	11	13
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441	1	2	4	5	6	7	8	10	11
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545	1	2	3	4	5	6	7	8	9
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633	1	2	3	4	4	5	6	7	8
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706	1	1	2	3	4	4	5	6	6
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767	1	1	2	2	3	4	4	5	5
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817	0	1	1	2	2	3	3	4	4
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857	0	1	1	2	2	2	3	3	4
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890	0	1	1	1	2	2	2	3	3
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916	0	1	1	1	1	2	2	2	2
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936	0	0	1	1	1	1	1	2	2
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952	0	0	0	1	1	1	1	1	1
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964	0	0	0	0	1	1	1	1	1
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974	0	0	0	0	0	1	1	1	1
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981	0	0	0	0	0	0	0	1	1
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986	0	0	0	0	0	0	0	0	0

Critical values for the normal distribution

If Z has a normal distribution with mean 0 and variance 1, then, for each value of p, the table gives the value of z such that

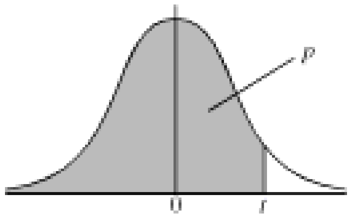
$$\mathrm{P}(Z \leqslant z) = p.$$

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

CRITICAL VALUES FOR THE *t*-DISTRIBUTION

If T has a t-distribution with ν degrees of freedom, then, for each pair of values of p and ν, the table gives the value of t such that:

$P(T \leqslant t) = p$.

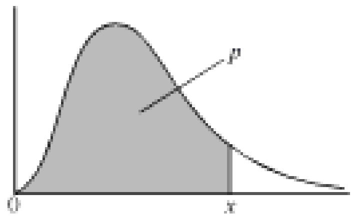


p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
$\nu = 1$	1.000	3.078	6.314	12.71	31.82	63.66	127.3	318.3	636.6
2	0.816	1.886	2.920	4.303	6.965	9.925	14.09	22.33	31.60
3	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.21	12.92
4	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.894	6.869
6	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.768
24	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.689
28	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.660
30	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

CRITICAL VALUES FOR THE χ^2-DISTRIBUTION

If X has a χ^2-distribution with ν degrees of freedom then, for each pair of values of p and ν, the table gives the value of x such that

$$P(X \leqslant x) = p.$$



p	0.01	0.025	0.05	0.9	0.95	0.975	0.99	0.995	0.999
$\nu = 1$	$0.0^3 1571$	$0.0^3 9821$	$0.0^2 3932$	2.706	3.841	5.024	6.635	7.879	10.83
2	0.02010	0.05064	0.1026	4.605	5.991	7.378	9.210	10.60	13.82
3	0.1148	0.2158	0.3518	6.251	7.815	9.348	11.34	12.84	16.27
4	0.2971	0.4844	0.7107	7.779	9.488	11.14	13.28	14.86	18.47
5	0.5543	0.8312	1.145	9.236	11.07	12.83	15.09	16.75	20.51
6	0.8721	1.237	1.635	10.64	12.59	14.45	16.81	18.55	22.46
7	1.239	1.690	2.167	12.02	14.07	16.01	18.48	20.28	24.32
8	1.647	2.180	2.733	13.36	15.51	17.53	20.09	21.95	26.12
9	2.088	2.700	3.325	14.68	16.92	19.02	21.67	23.59	27.88
10	2.558	3.247	3.940	15.99	18.31	20.48	23.21	25.19	29.59
11	3.053	3.816	4.575	17.28	19.68	21.92	24.73	26.76	31.26
12	3.571	4.404	5.226	18.55	21.03	23.34	26.22	28.30	32.91
13	4.107	5.009	5.892	19.81	22.36	24.74	27.69	29.82	34.53
14	4.660	5.629	6.571	21.06	23.68	26.12	29.14	31.32	36.12
15	5.229	6.262	7.261	22.31	25.00	27.49	30.58	32.80	37.70
16	5.812	6.908	7.962	23.54	26.30	28.85	32.00	34.27	39.25
17	6.408	7.564	8.672	24.77	27.59	30.19	33.41	35.72	40.79
18	7.015	8.231	9.390	25.99	28.87	31.53	34.81	37.16	42.31
19	7.633	8.907	10.12	27.20	30.14	32.85	36.19	38.58	43.82
20	8.260	9.591	10.85	28.41	31.41	34.17	37.57	40.00	45.31
21	8.897	10.28	11.59	29.62	32.67	35.48	38.93	41.40	46.80
22	9.542	10.98	12.34	30.81	33.92	36.78	40.29	42.80	48.27
23	10.20	11.69	13.09	32.01	35.17	38.08	41.64	44.18	49.73
24	10.86	12.40	13.85	33.20	36.42	39.36	42.98	45.56	51.18
25	11.52	13.12	14.61	34.38	37.65	40.65	44.31	46.93	52.62
30	14.95	16.79	18.49	40.26	43.77	46.98	50.89	53.67	59.70
40	22.16	24.43	26.51	51.81	55.76	59.34	63.69	66.77	73.40
50	29.71	32.36	34.76	63.17	67.50	71.42	76.15	79.49	86.66
60	37.48	40.48	43.19	74.40	79.08	83.30	88.38	91.95	99.61
70	45.44	48.76	51.74	85.53	90.53	95.02	100.4	104.2	112.3
80	53.54	57.15	60.39	96.58	101.9	106.6	112.3	116.3	124.8
90	61.75	65.65	69.13	107.6	113.1	118.1	124.1	128.3	137.2
100	70.06	74.22	77.93	118.5	124.3	129.6	135.8	140.2	149.4

WILCOXON SIGNED-RANK TEST

The sample has size n.

P is the sum of the ranks corresponding to the positive differences.

Q is the sum of the ranks corresponding to the negative differences.

T is the smaller of P and Q.

For each value of n the table gives the **largest** value of T which will lead to rejection of the null hypothesis at the level of significance indicated.

Critical values of *T*

	Level of significance			
One-tailed	0.05	0.025	0.01	0.005
Two-tailed	0.1	0.05	0.02	0.01
$n = 6$	2	0		
7	3	2	0	
8	5	3	1	0
9	8	5	3	1
10	10	8	5	3
11	13	10	7	5
12	17	13	9	7
13	21	17	12	9
14	25	21	15	12
15	30	25	19	15
16	35	29	23	19
17	41	34	27	23
18	47	40	32	27
19	53	46	37	32
20	60	52	43	37

For larger values of n, each of P and Q can be approximated by the normal distribution with mean $\frac{1}{4}n(n+1)$ and variance $\frac{1}{24}n(n+1)(2n+1)$.

WILCOXON RANK-SUM TEST

The two samples have sizes m and n, where $m \leqslant n$.

R_m is the sum of the ranks of the items in the sample of size m.

W is the smaller of R_m and $m(n + m + 1) - R_m$.

For each pair of values of m and n, the table gives the **largest** value of W which will lead to rejection of the null hypothesis at the level of significance indicated.

Critical values of *W*

	Level of significance											
One-tailed	0.05	0.025	0.01	0.05	0.025	0.01	0.05	0.025	0.01	0.05	0.025	0.01
Two-tailed	0.1	0.05	0.02	0.1	0.05	0.02	0.1	0.05	0.02	0.1	0.05	0.02
n	$m = 3$			$m = 4$			$m = 5$			$m = 6$		
3	6	–	–									
4	6	–	–	11	10	–						
5	7	6	–	12	11	10	19	17	16			
6	8	7	–	13	12	11	20	18	17	28	26	24
7	8	7	6	14	13	11	21	20	18	29	27	25
8	9	8	6	15	14	12	23	21	19	31	29	27
9	10	8	7	16	14	13	24	22	20	33	31	28
10	10	9	7	17	15	13	26	23	21	35	32	29

	Level of significance											
One-tailed	0.05	0.025	0.01	0.05	0.025	0.01	0.05	0.025	0.01	0.05	0.025	0.01
Two-tailed	0.1	0.05	0.02	0.1	0.05	0.02	0.1	0.05	0.02	0.1	0.05	0.02
n	$m = 7$			$m = 8$			$m = 9$			$m = 10$		
7	39	36	34									
8	41	38	35	51	49	45						
9	43	40	37	54	51	47	66	62	59			
10	45	42	39	56	53	49	69	65	61	82	78	74

For larger values of m and n, the normal distribution with mean $\frac{1}{2}m(m+n+1)$ and variance $\frac{1}{12}mn(m+n+1)$ should be used as an approximation to the distribution of R_m.

6 What else you need to know

This section is an overview of other information you need to know about this syllabus. It will help to share the administrative information with your exams officer so they know when you will need their support. Find more information about our administrative processes at **www.cambridgeinternational.org/eoguide**

Before you start

Previous study

We strongly recommend that learners starting this course should have studied, or be studying, Cambridge International AS & A Level Mathematics (9709) or the equivalent. See the introduction to section 3 of this syllabus for more details of expected prior knowledge.

Guided learning hours

We design Cambridge International AS & A Level syllabuses to require about 180 guided learning hours for each Cambridge International AS Level and about 360 guided learning hours for a Cambridge International A Level. The number of hours a learner needs to achieve the qualification may vary according to each school and the learners' previous experience of the subject.

Availability and timetables

All Cambridge schools are allocated to an administrative zone. Each zone has a specific timetable.

You can view the timetable for your administrative zone at **www.cambridgeinternational.org/timetables**

You can enter candidates in the June and November exam series.

Check you are using the syllabus for the year the candidate is taking the exam.

Private candidates can enter for this syllabus.

Combining with other syllabuses

Candidates can take this syllabus alongside other Cambridge International syllabuses in a single exam series. The only exceptions are:

- syllabuses with the same title at the same level.

Note that candidates can take AS & A Level Mathematics (9709) in the same exam series with AS & A Level Further Mathematics (9231).

Group awards: Cambridge AICE

Cambridge AICE (Advanced International Certificate of Education) is a group award for Cambridge International AS & A Level. It encourages schools to offer a broad and balanced curriculum by recognising the achievements of learners who pass exams in a range of different subjects.

Learn more about Cambridge AICE at **www.cambridgeinternational.org/aice**

Making entries

Exams officers are responsible for submitting entries to Cambridge International. We encourage them to work closely with you to make sure they enter the right number of candidates for the right combination of syllabus components. Entry option codes and instructions for submitting entries are in the *Cambridge Guide to Making Entries*. Your exams officer has access to this guide.

Exam administration

To keep our exams secure, we produce question papers for different areas of the world, known as administrative zones. We allocate all Cambridge schools to one administrative zone determined by their location. Each zone has a specific timetable.

Some of our syllabuses offer candidates different assessment options. An entry option code is used to identify the components the candidate will take relevant to the administrative zone and the available assessment options.

Support for exams officers

We know how important exams officers are to the successful running of exams. We provide them with the support they need to make entries on time. Your exams officer will find this support, and guidance for all other phases of the Cambridge Exams Cycle, at **www.cambridgeinternational.org/eoguide**

Retakes and carrying forward marks

Candidates can retake Cambridge International AS Level and Cambridge International A Level as many times as they want to. Information on retake entries is at **www.cambridgeinternational.org/retakes**

Candidates can carry forward the result of their Cambridge International AS Level assessment from one series to complete the Cambridge International A Level in a following series. The rules, time limits and regulations for carry-forward entries for staged assessment and carrying forward component marks can be found in the *Cambridge Handbook* for the relevant year of assessment at **www.cambridgeinternational.org/eoguide**

To confirm what entry options are available for this syllabus, refer to the *Cambridge Guide to Making Entries* for the relevant series.

Language

This syllabus and the related assessment materials are available in English only.

Accessibility and equality

Syllabus and assessment design

At Cambridge International, we work to avoid direct or indirect discrimination in our syllabuses and assessment materials. We aim to maximise inclusivity for candidates of all national, cultural or social backgrounds and candidates with protected characteristics, which include special educational needs and disability, religion and belief, and characteristics related to gender and identity. We also aim to make our materials as accessible as possible by using accessible language and applying accessible design principles. This gives all candidates the fairest possible opportunity to demonstrate their knowledge, skills and understanding and helps to minimise the requirement to make reasonable adjustments during the assessment process.

Access arrangements

Access arrangements (including modified papers) are the principal way in which Cambridge International complies with our duty, as guided by the UK Equality Act (2010), to make 'reasonable adjustments' for candidates with special educational needs (SEN), disability, illness or injury. Where a candidate would otherwise be at a substantial disadvantage in comparison to a candidate with no SEN, disability, illness or injury, we may be able to agree pre-examination access arrangements. These arrangements help a candidate by minimising accessibility barriers and maximising their opportunity to demonstrate their knowledge, skills and understanding in an assessment.

Important:

Requested access arrangements should be based on evidence of the candidate's barrier to assessment and should also reflect their normal way of working at school. This is explained in the *Cambridge Handbook* **www.cambridgeinternational.org/eoguide**

- For Cambridge International to approve an access arrangement, we will need to agree that it constitutes a reasonable adjustment, involves reasonable cost and timeframe and does not affect the security and integrity of the assessment.
- Availability of access arrangements should be checked by centres at the start of the course. Details of our standard access arrangements and modified question papers are available in the *Cambridge Handbook* **www.cambridgeinternational.org/eoguide**
- Please contact us at the start of the course to find out if we are able to approve an arrangement that is not included in the list of standard access arrangements.
- Candidates who cannot access parts of the assessment may be able to receive an award based on the parts they have completed.

After the exam

Grading and reporting

Grades a, b, c, d or e indicate the standard a candidate achieved at Cambridge International AS Level. 'a' is the highest and 'e' is the lowest grade.

Grades A*, A, B, C, D or E indicate the standard a candidate achieved at Cambridge International A Level. A* is the highest and E is the lowest grade.

'Ungraded' means that the candidate's performance did not meet the standard required for the lowest grade (E or e). 'Ungraded' is reported on the statement of results but not on the certificate. In specific circumstances your candidates may see one of the following letters on their statement of results:

- Q (PENDING)
- X (NO RESULT).

These letters do not appear on the certificate.

If a candidate takes a Cambridge International A Level and fails to achieve grade E or higher, a Cambridge International AS Level grade will be awarded if both of the following apply:

- the components taken for the Cambridge International A Level by the candidate in that series included all the components making up a Cambridge International AS Level
- the candidate's performance on the AS Level components was sufficient to merit the award of a Cambridge International AS Level grade.

On the statement of results and certificates, Cambridge International AS & A Levels are shown as General Certificates of Education, GCE Advanced Subsidiary Level (GCE AS Level) and GCE Advanced Level (GCE A Level).

How students, teachers and higher education can use the grades

Cambridge International A Level

Assessment at Cambridge International A Level has two purposes:

1 to measure learning and achievement

The assessment confirms achievement and performance in relation to the knowledge, understanding and skills specified in the syllabus.

2 to show likely future success

The outcomes help predict which students are well prepared for a particular course or career and/or which students are more likely to be successful.

The outcomes help students choose the most suitable course or career

Cambridge International AS Level

Assessment at Cambridge International AS Level has two purposes:

1 to measure learning and achievement

The assessment confirms achievement and performance in relation to the knowledge, understanding and skills specified in the syllabus.

2 to show likely future success

The outcomes help predict which students are well prepared for a particular course or career and/or which students are more likely to be successful.

The outcomes help students choose the most suitable course or career

The outcomes help decide whether students part way through a Cambridge International A Level course are making enough progress to continue

The outcomes guide teaching and learning in the next stages of the Cambridge International A Level course.

School feedback: 'Cambridge International A Levels are the 'gold standard' qualification. They are based on rigorous, academic syllabuses that are accessible to students from a wide range of abilities yet have the capacity to stretch our most able.'

Feedback from: Director of Studies, Auckland Grammar School, New Zealand

Changes to this syllabus for 2026 and 2027

The syllabus has been updated. This is version 1, published September 2023.

You must read the whole syllabus before planning your teaching programme. We review our syllabuses regularly to make sure they continue to meet the needs of our schools. In updating this syllabus, we have made it easier for teachers and students to understand, keeping the familiar features that teachers and schools value.

Any textbooks endorsed to support the syllabus for examination from 2020 are still suitable for use with this syllabus.

School feedback: 'While studying Cambridge IGCSE and Cambridge International A Levels, students broaden their horizons through a global perspective and develop a lasting passion for learning.'

Feedback from: Zhai Xiaoning, Deputy Principal, The High School Affiliated to Renmin University of China

Cambridge Assessment International Education, The Triangle Building, Shaftesbury Road, Cambridge, CB2 8EA, United Kingdom

t: +44 (0)1223 553554 email: info@cambridgeinternational.org www.cambridgeinternational.org